CBCS SCHEME

USN | | | | | | | | | | 18MAT21

Second Semester B.E. Degree Examination, July/August 2021
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

1 a. Find the divergence and curl of the vector, $\vec{V} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$ at the point (2, -1, 1). **(06 Marks)**

b. Find the workdone in moving a particle in the force field $F = 3x^2i + (2xz - y)i + zk$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from x = 0 to x = 2. **(07 Marks)**

c. Evaluate the surface integral $\iint_S \vec{F}.\vec{N}ds$ where $F = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region $x^2 + y^2 = 4$, z = 0 and z = 3. **(07 Marks)**

2 a. Find Curl (Curl $\vec{A}$) where $\vec{A} = x^2yi - 2xzj + 2yzk$ at the point (1, 0, 2). **(06 Marks)**

b. If $\vec{u} = x^2i + y^2j + z^2k$ and $\vec{v} = yzi + zxj + xyk$, show that $\vec{u} \times \vec{v}$ is solenoidal. **(07 Marks)**

c. Evaluate $\int_C (\sin zdx - \cos xdy + \sin ydx)$ by using Stoke's theorem, where C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$ and z = 3. **(07 Marks)**

3 a. Solve: $(D^4 - 1)\, y = 0$ **(06 Marks)**

b. Solve: $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = \sinh(2x + 3)$ by Inverse differential operator method. **(07 Marks)**

c. Solve: $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \log x$ **(07 Marks)**

4 a. Solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1 - e^x)^2$ **(06 Marks)**

b. Solve: $(D - 2)^2y = 8\,(e^{2x} + x + x^2)$ by Inverse differential operator method. **(07 Marks)**

c. A particle moves along the x-axis according to the law $\frac{d^2x}{dt^2} + \frac{6dx}{dt} + 25x = 0$. If the particle is started at x = 0 with an initial velocity of 12ft/sec to the left, determine x(t). **(07 Marks)**

5 a. Form the partial differential equation by eliminating the arbitrary constants in $(x - a)^2 + (y - b)^2 = z^2 \cot^2 \alpha$, where α is the parameter. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \text{Sinx Siny}$ for which $\frac{\partial z}{\partial y} = -2\text{Siny}$ when x = 0 and z = 0 if y is an odd multiple of π/2. **(07 Marks)**

c. Derive one dimensional heat equation. **(07 Marks)**

6 a. Form the partial differential equation by eliminating the arbitrary functions from $Z = f(x + at) + g(x - at)$. (06 Marks)

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$. (07 Marks)

c. Solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x + y)u$, by the method of separation of variables. (07 Marks)

7 a. Find the nature of the series $\sum_{n=1}^{\infty} a^{n^2} x^n$, $a < 1$ (06 Marks)

b. Prove that: $J_{Y_2}(x) = \sqrt{\frac{2}{\pi x}}\,\text{Sin}x$ (07 Marks)

c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$ find the values of a, b, c, d. (07 Marks)

8 a. Test for convergence the series,

$$\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \ldots\ldots$$ (06 Marks)

b. Express $x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. (07 Marks)

c. Show that i) $P_2(\cos\theta) = \frac{1}{4}(1 + 3\cos 2\theta)$ ii) $P_3(\cos\theta) = \frac{1}{8}(3\cos\theta + 5\cos 3\theta)$. (07 Marks)

9 a. From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46. (06 Marks)

Age	45	50	55	60	65
Premium (In Rupees)	114.84	96.16	83.32	74.48	68.48

b. Find cube root of 37 correct to 3 decimal places, using Newton-Raphson method. (07 Marks)

c. Use Simpson's 1/3rd rule to find $\int_0^{0.6} e^{-x^2}\,dx$ by taking 6 sub-intervals. (07 Marks)

10 a. Using Newton's backward Interpolation formula, find the interpolating polynomial function given by the following table:

x	10	11	12	13
f(x)	22	24	28	34

(06 Marks)

b. Find a Real Root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places using Regula Falsi method. (07 Marks)

c. Evaluate $\int_0^1 \frac{x\,dx}{1 + x^2}$ by Weddle's rule taking seven ordinates. (07 Marks)

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